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Peculiarities of dusty plasma space distribution in traps

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Abstract
The distributions of dust particle density in the various types of electromagnetic and gravitational traps are discussed to clarify the specific behaviour of these distributions for changing parameters of plasma. Correlations between the dust particles (Yukawa interaction potential) are taken into account by the use of the density functional formalism for finite temperatures in electro-gravitational and spherical traps. Because the volume and in many cases the number of particles (with change of the external conditions) in the traps are undetermined quantities, we suggest a possible determination for the average density in traps.

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1. Introduction

The behaviour of the various objects in a finite volume or in a restricted space is a fascinating problem for physics. Usually particles interact with one another and this interaction is essential for their behaviour and, in particular, for the density distribution. The typical examples in physics are the hot plasma particles in stellarators or in tokamaks [1, 2], dusty plasmas in electromagnetic and gravitational traps [3–5], and ultracold Bose and Fermi gases [6, 7]. In the present paper we focus on the relatively simple physical systems, where dust particles interact by some short-range potential (e.g., Yukawa potential) and are placed in some confining external field, which can be formed by a combination of electromagnetic and gravitational potentials. Depending on interaction potential, trap properties and the external parameters the dust particles can be in ordered or disordered states. An essential interest has recently been attracted by traps, in which dusty plasma consisting of two- or three-dimensional shells (Yukawa balls, e.g., [8, 9]), or voids (e.g., [10, 12]) are formed. Now the density distribution of these objects has been well investigated [13, 14]. Recently, the ground state of an externally confined one-component Yukawa plasma was numerically and analytically investigated for a spherical trap at zero temperature [15]. The spherically decreasing density with a finite distance to the border has been established.

2. One-dimensional electro-gravitational confinement for charged particles

Let us consider a one-dimensional inhomogeneity of the particle density in the vertical direction (z-direction) under the influence of an electrical field $E(z)$ and gravitational field g. We choose the electric field along the z-axis in a form, which permits analytical integration

$$E(z) = -E_0 \frac{s_0 z + t_0}{1 + pz^2}. \tag{1}$$

The minus in (1) shows that this field is directed downwards. Let us consider now the distribution of the dust particles $n(z)$ in the electro-gravitational trap formed by the electrical φ and gravitational Mgz potentials. If interaction between the particles is absent this distribution is the Boltzmann one

$$\begin{aligned} n(z) \equiv \exp[-U_{\mathrm{eff}}/T] &= C \exp\left[-\frac{1}{T}(Mgz - \varphi(z))\right] \\ &= C \exp\left\{-a\left[u - bt_0 \cdot \arctan(u) + \frac{bs_0 z_1}{2} \ln(1+u^2)\right]\right\}, \end{aligned} \tag{2}$$

where C is the normalization constant, T is the temperature of the dust particles and we introduced the designations $z_1 \equiv 1/\sqrt{p}$, $a = Mgz_1/T$, $b = QeE_0/Mg$, $t_0 = |E(0)/E_0|$ and $u = \sqrt{p}z \equiv z/z_1$. The trap can effectively confine the particles if $(F_E)_{\max} > Mg$. The relation between the forces in the model under consideration and the effective potential has been calculated for fixed parameter values of a and t. The parameter t characterizes the electric field at the bottom of the column, when t decreases the field at the bottom also decreases and the particles can drop to the bottom where the second pit appears.

Let us determine the 'average density' in the considered cases. There is no unique way for the determination of the 'average density'. If there is only one pit we can introduce for this purpose the concept of the 'essential volume', namely the 'volume' V_μ, in which the main part of the particles $N_\mu = \mu N$ is concentrated (lets say with $\mu = 0.95$, $N_\mu = 0.95N$). Then the 'average density' is equal to $n_\mu = N_\mu/V_\mu$. To find V_μ (in the one-dimensional case under consideration z_μ) we have to find the solution z_μ of the equation

$$\int_0^{z_\mu} n(z)\,\mathrm{d}z = N_\mu, \tag{3}$$

where $n(z)$ is the normalized density. For $N_\mu = 950$ ($N = 1000$, $\mu = 0.95$) and $t = 0.5$ for the values $a = 4; 5; 6$ the solutions of equation (3) have been obtained. For the respective set of the values a the set of solutions z_μ of equation (3) are $z_\mu = 3.51; 3.28; 3.14$. Calculations [20] show that density n_μ increases when the particle temperature T decreases (a increases, as is shown in a figure given in [20]).

For the particle density with Yukawa interaction in the electro-gravitational trap ($Y \equiv n(z)/n(0)$; $\zeta \equiv 4\pi Q_0^2 n(0)/\kappa^2 T$, where Q_0 and κ are the charge of the particles and the Debye radius of the interaction potential) we find the nonlinear equation

$$Y = \exp\left\{-a\left[u - bt_0 \cdot \arctan(u) - \frac{bs_0 z_1}{2}\ln(1+u^2)\right] - \zeta Y + \zeta\right\}. \tag{4}$$

The value $n(0)$ is determined by the normalization condition. The numerical results for the interacting particles in electro-gravitational trap will be presented with the respective figures in a more extended paper [20].

3. Confinement for plasma with Yukawa interaction in a parabolic pit at finite temperature

To consider the problem of trapped particles with interaction we use the density-functional formalism [17] at finite temperatures [18] in the simple model [19]. Let us choose the confining potential $\Phi(r) = \alpha r^2/2$ as parabolic. We find a solution which has a spherical symmetry. Then the minimization of the potential Ω leads to the equation for the density

$$X = \exp[-q + \beta - \beta X], \tag{5}$$

where we use the notations $X = D(r)/D(0) \equiv n(r)/n(0), D(r) = n(r)/n_c, n_c = 3\alpha/4\pi Q^2, q = l^2/2t \equiv \alpha r^2/2T, \beta = 3D(0)/ft \equiv 3n(0)\alpha d_c/\kappa n_c T, f = \kappa d_c, d_c = (2Q^2/\alpha^{1/3}), l = r/d_c, s = n_c\lambda^3, t = T/\alpha d_c^2$. For $f = 1$ and four values of $\beta_i = 0.02; 0.5; 5; 15$ we find for the convenient variable $\tau_f \equiv f t^{5/2}$ the respective values $(\tau_f)_i = 1.969 \times 10^4; 5.943 \times 10^2; 1.383 \times 10^1; 1.245$ ($t_i = (\tau_f)_i^{2/5}$ for the case $f = 1$). For lower values of the dimensionless temperatures $(\tau_f)_i$ the curves $X(l)$ decrease rapidly and the density distribution $n(r)/n_c$ becomes narrower.

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